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# Seismic Design of Ribbed Foundation Plates

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SYNOPSIS Circular foundation plates of variable thickness reinforced by ring ribs and resting on an elastic subgrade subjected to an action of seismic loads are investigated. Rigorous solutions of the posed problems are obtained.

## INTRODUCTION

Foundation slabs of television and water towers, chimneys, bottom of tanks are often designed as circular or annular plates of nonuniform thickness. For their seismic analysis the consideration of antisymmetric loading $Q=Q\sin\theta$ and $M=M\sin\theta$ including discontinuous loads, as it have been shown by Koreneva (1992), is of great importance. This paper treats seismic analysis of ribbed isotropic and orthotropic plates with varying thickness resting on an elastic isotropic homogeneous semi-infinite medium or on Winkler's basis. Antisymmetric bending of circular plate of constant thickness reinforced by ring ribs without taking into account of an influence of elastic subgrade was examined by Reut et al. (1977). In the work by Koreneva have been studied the symmetric bending of ribbed circular plate with power thickness on Winkler's basis. Under symmetric loading ribs creating near rigid plate supporting along a ring contour. In these cases ring ribs are undergoing pure bending. For unsymmetric loading bending moments, as well as twisting moments, are to be considered. Therefore, an efficiency of ring ribs application for plates subjected to an action of antisymmetric loads is to be discussed. The present work gives rigorous solutions of the abovementioned problems in closed form by using analytical techniques. This paper covers the new results received by the author and points the way of hers previous works utilization for seismic analysis of ribbed foundation plates.

## ANALYSIS

The peculiarity of the proposed work is the fact that magnitudes of twisting moments are small and may be neglected for the study of fairly rigid rings. For such cases ring ribs can be applied profitably. Ring ribs are considered as cylindrical shells. A contact of a plate of variable thickness resting on an elastic subgrade and a shell is examined. There are no any deformations in the middle planes of the plates with the thicknesses under study. Circumferential displacement of an edge of a shell vanishs. Because of this all the radial stresses may be expressed by radial bending moments $M_r$. So, within the framework of the adopted assumption, only magnitudes of amplitude radial bending moments are to be determined; for this analysis the equations of the method of forces are used:

$$\sum_{j=1}^{n} \delta_{ij} X_j + \Delta_{iq} = 0, \qquad (i=1,2,\dots n) \qquad (1)$$

where $X_j$ -is an amplitude magnitude of an unknown generalized force applied along the circumference with the radius $a_j$, corresponding to the rib "j"; coefficients $\delta_{ij}$ are

$$\delta_{ij} = \bar{\delta}_{ij} + \bar{\bar{\delta}}_{ij} + \bar{\bar{\bar{\delta}}}_{ij}, \text{ when } i=j$$

$$\delta_{ij} = \bar{\delta}_{ij} + \bar{\bar{\delta}}_{ij} \quad , \text{ when } i \neq j$$

where $\bar{\delta}_{ij}$ is an amplitude value of an angular displacement of the plate at "i" induced by unit generalized force at "j"; $\bar{\bar{\delta}}_{ij} = w_{ij}/r_i$; $w_{ij}$ - is an amplitude value of the plate's deflection at "i" caused by a single generalized force at "j", $r_i$ -radius of the rib "i"; $\bar{\bar{\bar{\delta}}}_{ij}$ (i=j)-is an amplitude value of an angular displacement of the edge of the shell ( at the place of it's connection with the plate) produced by single generalized force at "i".

Term $\Delta_{iq}$ is

$$\Delta_{iq} = \bar{\Delta}_{iq} + \bar{\bar{\Delta}}_{iq},$$

where $\bar{\Delta}_{iq}$ -an amplitude value of an angular displacement of the plate in "i" induced by applied loads; $\bar{\bar{\Delta}}_{iq} = w_{iq}/r_i$, $\bar{\bar{\Delta}}_{iq}$ is an amplitude value of a deflection of the plate at "i" caused by applied loads.

Calculating generalized forces from the system (1), we can determine magnitudes of deflections, slopes, bending moments and shearing forces.

For the study of the problems formulated above, the action of discontinuous loading of the plate by moments $m=m_0\sin\theta$ and forces $q=q_0\sin\theta$, distributed along circles non-coinciding with a contour is to be considered.

Let us examine antisymmetric bending of a plate with the parabolic rigidity:

$$D=D_0(1-x), \quad x=(r/r_0)^2, \qquad (2)$$

where $D_0, h_0$ are constants.
Homogeneous governing equation for this case is

$$x^4(I-x)\frac{d^4w}{dx^4}+2x^3(2-3x)\frac{d^3w}{dx^3}+\frac{x^2}{2}(3-(I2+\nu)x)\frac{d^2w}{dx^2}-$$
$$-\frac{\nu x^2}{2}\frac{dw}{dx} - \frac{(3+(3-2\nu)x)w}{16}=0, \qquad (3)$$

where w-deflection, $\nu$ -Poisson's ratio.
The general solution of (3) is represented in the following form:

$$w(x,\theta)=(A_1w_1(x)+A_2w_2(x)+A_3w_3(x)+A_4w_4(x))\sin\theta \qquad (4)$$

$w_1=x^{1/2}$, $w_2=x^{-1/2}$, $w_3=x^{1/2}y_1(x)$, $w_4=x^{1/2}y_2(x)$, where $y_1=F(\frac{I}{2}+\delta, \frac{I}{2}-\delta; 2; x)$,

$y_2=\Phi(\frac{I}{2}+\delta, \frac{I}{2}-\delta; 2; x)$-are hypergeometric functions, satisfying the hypergeometric equation

$$x(I-x)\frac{d^2y}{dx^2}+2(I-x)\frac{dy}{dx}+\frac{I-\nu}{2}y=0, \qquad (5)$$

$\delta=0,5\sqrt{I+2(I-\nu)}$, $A_1, A_2, A_3, A_4$-constants.

For the investigation of discontinuous loading Cauchy functions $Z_1(a;x), Z_2(a;x), Z_3(a;x), Z_4(a;x)$, possesing certain properties, are to be determined. First, it is necessary to obtain Wronskian $W(x)$ for the solutions (4). Calculating of $W(x)$ is very bulky. Here is the Wronskian for hypergeometric equation $W_g=Cx^{-2}$, $C=2/(3+\nu)$ and the following functional determinants

$$W_{1g}=-2Cx^{-2}, \quad W_{2g}=\frac{\beta C}{I-x}x^{-3},$$
$$W_{3g}=\frac{\beta Cx^{-4}}{(I-x)}(4x-3),$$
$$W_{4g}=-\frac{\beta Cx^{-4}}{(I-x)^2}(2-\beta), \quad \beta=\frac{I-\nu}{2}$$

which have to be used during the process of our analysis.
Then, we receive

$$W(x)=-\beta^2C/(x^4(I-x)^2)$$

The similar result is obtained by using of Liouville's formula.

Further, Cauchy functions have been determined. Two of them are given below

$$Z_3(a;x)=\frac{2a}{C(I-\nu)}\Big\{-a^{-1/2}C(5a-3)x^{1/2}+a^{1/2}C(4a-3)x^{-1/2}-$$
$$-a^{1/2}(I-a)\Big[ay_2'(a)-\frac{I}{I-a}y_2(a)\Big]x^{1/2}y_1(x)+$$
$$+a^{1/2}(I-a)\Big[ay_1'(a)-\frac{I}{I-a}y_1(a)\Big]x^{1/2}y_2(x)\Big\},$$
$$Z_4(a;x)=\frac{2a^2(I-a)}{C(I-\nu)}\Big\{Ca^{1/2}x^{1/2}-Ca^{1/2}x^{1/2}+$$
$$+a^{1/2}y_2(a)x^{1/2}y_1(x)-a^{1/2}x^{1/2}y_1(a)y_2(x)\Big\} \qquad (6)$$

The expression for the deflection of the plate with the rigidity (2) with the clamped inner boundary is:

$$w(x,\theta)=w_1(x)\sin\theta=$$
$$=\Big[\frac{M_0r_0^2}{D(a)}Z_3(a;x)-\frac{Q_0r_0^3}{D(a)}Z_4(a;x)\Big]\sin\theta, \qquad (7)$$

where a-radius of an inner boundary, $M_0, Q_0$-the bending moment and the shearing force when x=a. For the plate loaded by moments $M_i\cos\theta$, distributed along the circle with the radius $a_i$, and by forces $P_j\cos\theta$, distributed along the circumference with the radius $a_j$, we arrive at the following expression for deflections:

$$w(x,\theta)=\Big[w_1(x)-\frac{M_ir_0^2}{D(a_i)}Z_3(a_i;x)+$$
$$+\frac{Q_jr_0^3}{D(a_j)}Z_4(a_j;x)\Big]\cos\theta \qquad (8)$$

In the similar manner the expression for slopes may be written.
For the study of a plate and a ring rib interaction the foregoing discontinuous solutions are used; for the determination of unknown generalized forces $X_j$ from the system (I) we have to consider the plate of variable thickness subjected to an action of moments $m=m_0\sin\theta$, distributed along circles non-coinciding with the contour. For the determination of displacements of the plate we have to use formulae (6)-(8). Formulae for calculation of contact stresses and displacements in a shell are given in the book by Chernina (I968). Taking into account an influence of elastic, isotropic semi-infinite medium, the method, described in the work by Koreneva (I994), is applied. Reactions of a subgrade are replaced by reactions of simple supports, arranged along circles non-coinciding with a contour, between the plate and the basis. Theplate is divided into rings of an arbitrary and indential width. The flexure of the construction is simultaneously induced by reactions of supports of the mentioned primary system and by applied loads. Here we also use the solutins (8). For the consideration of simple supports reactions Cauchy functions $Z_4(a;x)$ are utilized. Now calculated bending moments are to be determined.

## CONCLUSIONS

With the results of the works by Koreneva (I992, I987) we can obtain in the similar way solutions for ribbed plates of power thickness on Winkler's foundation, as well, as for a plate of linear thickness on an elastic semi-infinite medium.
This work receives rigorous solutions in terms of mathematical functions; the effective contemporary methods for their computation-expansions of mathematical functions in series of Chebychev polynomials of the first kind and rational approximations of the Padé class are used. The outlined methods have been given in the works

by Luke,for example in (1977).
This paper covers rather complicated problems of circular foundation slabs of variable thickness design.The results cited in this work are the base for the further seismic analysis of foundation plates.

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